Predicting Bridge Elements Deterioration, using Collaborative Gaussian Process Regression ⋆

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Abstract: Roadway and railway bridges are not only integral, but also vulnerable parts of terrestrial transport networks. Structural failures of bridges may lead to disastrous consequences on users and society at large. Bridge predictive deterioration models are extremely important for effective maintenance decision-making. However, the lack of enough inspection data between maintenance activities of a bridge complicates the development of accurate predictive models. Presented herein is a Gaussian Process Regression (GPR) based collaborative model for predicting the condition of bridge elements with limited available inspection data per bridge. This model has been applied in 137 bridge decks, showing that collaborative prognosis has the potential to predict the condition of different types of bridge elements, composing different types of bridges.

Keywords: Transportation infrastructure, asset management, bridge maintenance, stochastic model, collaborative prognosis

1. INTRODUCTION

The economic development of a country is linked with the available resources to society and the effectiveness of their use (Ivanová and Masárová, 2013). Transport networks contribute to modern society's daily activities by serving mobility and productivity (Chan et al., 2010). The most widely used modes of transport are the terrestrial, constituting of roadways and railways, while representing 5158 billion passenger-kilometres in 2015 in EU-28 (Eurostat, 2017). Bridges are an integral and at the same time vulnerable element of terrestrial transportation networks. Structural failures of bridges, which are primarily located at intersections of highways/railways, can lead to catastrophic consequences not only on users but also on the society at large. Probable extensive effects consist of traffic rerouting, productivity reduction, loss of access to areas of interest, as well as increment of travel time, distance and subsequently of carbon monoxide emissions and environmental pollution.

Railway and highway bridges gradually deteriorate over their lifetime. A variety of extreme events, composed of man-made events (e.g. terrorist attacks, bridge strikes) and natural disasters (e.g. floods, earthquakes), as well as heavy traffic and insufficient maintenance, can dramatically accelerate their deterioration (Zhang and Wang, 2017). Only in the U.S., the number of bridges is 614,387 bridges, with around 40% of them being over 50 years old and 9.1% being structurally deficient, while 188 million trips/day are conducted over structurally deficient bridges. Additionally, their average age is continuously rising, while many bridges are close to the end of their design lifetime. A recent approximation of money required for bridge rehabilitation equals $123 billion (ACSE, 2017). Ensuring that bridges can operate under normal and extreme conditions requires frequent inspections and maintenance, when needed, to meet safety threshold values. Transportation departments often face the challenge of managing thousands of bridges, having budgetary constraints and no systematic way for deciding on the optimal timing for repairing bridge elements.

The progress in infrastructure condition assessment and prediction using sensors, such as vibrometers and cameras, as well as in data analysis methods in recent years (Chuang et al., 2019; Hadjidemetriou and Christodoulou, 2019; Hadjidemetriou et al., 2018; Malekjafarian et al., 2018) has motivated researchers and practitioners to evaluate the benefits of asset predictive maintenance, compared to reactive maintenance. Thus, there is an increased interest in predictive maintenance prioritisation of multi-system multi-component networks (MSMCN). MSMCN are networks consisting of numerous systems which are in turn composed of several components. A bridge can be examined as a system of multiple components (e.g. primary deck element), whilst belonging in a network of multiple bridges. Predictive maintenance, which is the decision-making for maintenance or replacement processes based on predictive models, has the potential to enhance reliability and decrease maintenance cost of systems and networks

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(Mobley, 2002). The accuracy of predictive deterioration models becomes extremely important since it is directly correlated with the effectiveness of maintenance decision-making.

Predicting bridge elements deterioration becomes particularly challenging due to the diversity of bridge features and operating environments, as well as the lack of data. Bridge features may consist of type, material, usage and size (such as dimensions and number of spans), while operating environment characteristics include location, traffic and weather. In addition, most transport infrastructure owners have been collecting and storing bridge condition and maintenance data only for the last few decades, while their inspection rate varies between one to seven years.

With the aforementioned in mind, the following section of the paper describes the state of research in deterioration models for assets in general and specifically bridges, the identified gap in knowledge, and the objective of the present study. The sections that follow present the proposed methodology and a conducted case study. Lastly, findings, conclusions, and future research are discussed.

2. BACKGROUND

Overcoming the barriers of modelling deterioration of infrastructure assets and specifically of bridges or bridge components has attracted the interest of researchers. A few of them formed empirical relationships between bridge condition indices and the attributes affecting deterioration of bridges. These deterministic models assume that the relationship between bridge condition and time is certain, ignoring the randomness and uncertainty in deterioration procedure. Thus, if the input variables do not change, the prediction remains the same (Kotze et al., 2015). Wu et al. (2017) further categorised these predictive models as utilising straight-line extrapolation, regression, and curve-fitting methods (Chen et al., 2010; Morcous et al., 2002; Wei and Liu, 2013).

Another approach worth mentioning concentrates on deterioration mechanisms of bridge elements and on bridge reliability regarding strength limit states. Either deterministic mathematical equations or Monte Carlo Simulation (MCS) are used for the deterioration models. One of the initial examples of this group of studies is the work done by Frangopol et al. (1997), who introduced an optimisation method for scheduling the lifetime inspection and maintenance of bridges. At a later stage, Estes and Frangopol (1999) improved the previous work by proposing an optimisation system-based approach that attempts to balance expected life-cycle cost and lifetime reliability. Bocchini et al. (2011) extended this approach from the bridge-level to the level of network of bridges, while proposing a random field-based method to improve the efficiency of life-cycle analysis under uncertainty of bridge networks.

Another group of studies considers the probabilistic nature of bridge deterioration, and thus uses stochastic predictive models. Most stochastic models are based on Markov chain theory. For instance, Kleiner (2001) modelled asset deterioration as a semi-Markov procedure that is discretised into condition states. The waiting time in every state is assumed to be a random variable with a probability distribution. Another work by Thompson and Johnson (2005) analysed California bridge condition data set to estimate the deterioration transition probabilities and to examine if the main assumptions of Markov chain-based models for bridge deterioration can be validated. Similarly, Puz and Radic (2011) utilised homogeneous Markov procedures with a continuous parameter (i.e. time) and a finite set of condition states to probabilistically predict structure condition. The continuous parameter allows the calculation of probabilities of each condition state in any moment. In addition, Ranjith et al. (2013) used a stochastic Markov chain model to predict the condition of timber bridge elements and tested it on inspection data from the Roads Corporation of Victoria, Australia. The research work of Wellalage et al. (2015) proposed a Metropolis-Hasting algorithm-based Markov chain MCS method to overcome the limitation of existing nonlinear optimisation-based algorithms that fail to identify the optimum transition probability matrix values, leading to invalid predictions. Finally, Chang et al. (2019) presented a stochastic deterioration methodology that combines logistic regression, Markov chains and classification trees.

Another notable approach is the application of artificial intelligence and machine learning on asset deterioration models. For example, Lee et al. (2012) proposed a data processing technique for backward prediction model outcomes, by filtering out condition ratings for long-term deterioration prediction using Time Delay Neural Network. This research work showed potential to enhance accuracy of current bridge deterioration models that are based on artificial intelligence. Furthermore, Callow et al. (2013) used a hybrid optimisation technique to remove meaningless condition ratings as input for long-term prediction modelling, improving a computational costly procedure of neural network. Last, Galal Ali et al. (2019) used artificial neural networks and data from Missouri, USA to predict the condition of long span bridges.

Summarising, although there are multiple bridge deterioration models, there is a research gap in handling the limited amount of available historical inspection data per bridge, as well as the diversity of bridges and bridge elements in terms of attributes and the environment that they are exposed to. Given this, the current paper aims to develop a methodology that can be applied to any type of bridge element, especially when data is limited.

3. METHODOLOGY

Presented herein is a prediction model for failures in bridge elements, using collaborative Gaussian Process Regression (GPR). GPR is a non-parametric and data driven regression technique, which generates a stochastic distribution of functions mapping the inputs to corresponding outputs for a given dataset. A major benefit of GPR is that it can quantify the confidence of the predictions (Alvarez et al., 2011). Applications of GPR include Lithium ion battery health estimation (Richardson et al., 2017) and learning the dynamics of robotic arms (Bocsi et al., 2011).

GPR is selected here for bridge condition prediction because of the low frequency of bridge inspections. Since the bridges are inspected once in several years, their deterioration process cannot be continuously tracked. Moreover, the bridges undergo timely maintenance activities and thus uninterrupted inspection records from their new to failed states are rare. GPR can extrapolate such scattered inspection information to unrecorded condition, and of generating a distribution of functions that describe bridge deterioration throughout their lifetime.

GPR assumes a joint multivariate normal distribution for all the outputs in dataset. The output for any given input data point is the marginal normal distribution at that point. The marginal distribution for each input point is Gaussian, characterised by its mean and standard deviation. The mean is the predicted value of the output, and the standard deviation is a measure of the prediction's confidence. A higher standard deviation implies lower confidence. The marginal distributions for unknown points are predicted based on their similarity with the known points from the training dataset. Depending on the application, similarities are evaluated using various kernel functions which show large values for points lying closer to one another and small values for those far apart (Rasmussen, 2004).

The mathematical description presented here is extracted from Rasmussen (2004). For the input space X, the corresponding function is estimated as $f : X \rightarrow \mathcal{R}$ from the input space to the reals. f is a Gaussian process if for any vector of inputs $x = [x_1, x_2, ..., x_n]^T$ such that $x_i \in X$ for all i, the vector of outputs $f(x) = [f(x_1), f(x_2), ..., f(x_n)]^T$ is Gaussian distributed. GPR is specified by a mean function $\mu : X \rightarrow \mathcal{R}$, such that $\mu(x)$ is the mean of $f(x)$ and a covariance, or kernel, function $k : X \times X \rightarrow \mathcal{R}$ such that $k(x_i, x_j)$ is the covariance between $f(x_i)$ and $f(x_j)$. We say $f \sim GP(\mu, k)$ if for any $x_1, x_2, ..., x_n \in X$, $[f(x_1), f(x_2), ..., f(x_n)]^T$ is Gaussian distributed with mean $[\mu(x_1), \mu(x_2), ..., \mu(x_n)]^T$ and $n \times n$ covariance matrix K_{xx}:

$$\begin{bmatrix} k(x_1, x_1) & k(x_1, x_2) & ... & k(x_1, x_n) \\ k(x_2, x_1) & k(x_2, x_2) & ... & k(x_2, x_n) \\ ... & ... & ... & ... \\ k(x_n, x_1) & k(x_n, x_2) & ... & k(x_n, x_n) \end{bmatrix}$$

GPR sequentially evaluates the covariance for neighbouring points using a kernel function, followed by calculating their corresponding marginal distributions. As the granularity of neighbouring unknown points is increased, it approaches a continuous domain and eventually is equivalent to a function with domain of all possible input values. Despite poor scalability because of the computations involved, GP models can be optimised to achieve a trade-off between fitting the data and smoothing. GPR is therefore a favourite solution for problems with small regression datasets (Chapados and Bengio, 2008). Our methodology for bridge elements prediction can be separated into the following main phases: (i) clustering of similar bridges and pooling of their data together; and (ii) application of GPR for fitting functions to the data.

Clustering similar bridges (phase 1) leads to a more descriptive record. For the reasons explained before in this section, a single bridge would not have enough data describing its deterioration. The features governing bridge deterioration are identified and used as the basis for clustering similar bridges. Such features can be intrinsic like bridge material, mileage, or span count, or extrinsic like local weather conditions or traffic. Our hypothesis is that bridges with common features are bound to deteriorate similarly. Such collaborative deterioration modelling has been proposed as a solution and proved useful to the problem of lack of local data in recent literature (Palau et al., 2018). Within a cluster, the inspection records corresponding to different ranges of bridge condition are concatenated together and a common training dataset is attained. This dataset consists of time-series of inspections ranging from best inspected condition in the cluster to the worst.

Next (Phase 2), GPR is used to predict values for unknown data points. Since the bridge condition deteriorates over time, we need a decreasing prior mean value function for the GPR priors. This is different from conventional applications where the mean is usually zero, or a constant value. The mean value for regression is calculated by fitting a straight line using a least square fit. Using this mean value and random covariances, a prior distribution of functions is generated. This prior is updated according to the data from the previous step, with exponential kernel (1) calculating covariances for unknown data points. Exponential kernel is often the default kernel for GP applications because of its universal nature, the possibility of integrating it against most functions, infinite possible priors, and its only two governing parameters which can be easily tuned to suit for the given application (Duvenaud, 2014).

$$k(x_a, x_b) = \sigma^2 \exp\left(\frac{-(x_a - x_b)^2}{2l^2}\right) \qquad (1)$$

Where,

l = characteristic length

σ^2 = signal variance

(x_a, x_b) = points for which covariance is calculated

4. CASE STUDY

The proposed methodology was applied on a real-world bridge inspection dataset. This dataset was provided by a large transport infrastructure owner in the UK. The organisation name cannot be disclosed for confidentiality reasons. Dataset consists of inspection records of several bridges across the UK maintained by the organisation. Bridges undergo regular inspections every three or four years, and they are recommended for maintenance if the condition is considerably degraded. Each bridge component is rated independently based on the infrastructure owner's internal specifications. The condition index used in this study ranges from 100 to 0, with 100 indicating perfect condition. Different infrastructure owners use different condition indices. However, the presented prediction methodology is applicable to any bridge element, rated by any infrastructure owner. The case study, which serves as an example application, focuses on bridge deck elements.

Before proceeding to the clustering and GPR phases, bridges, which have deteriorated without being interrupted by any maintenance activities, were selected. These are the bridges that accurately resemble the deterioration process, and thus they were used for the regression model. Such bridges are characterised by consecutive inspections, where the bridge condition either deteriorated or remained constant. The final cleaned dataset for the current case study comprised of total 137 independent deck elements, and 295 data points representing various stages of decks' lifecycles.

The selected data points were separated into four clusters based on bridge deck material (Phase 1). Bridge engineers of the organisation, which provided the data, recommended that bridge deck material is the major influencer amongst the available features. Analyses for the two clusters with the highest number of data points (71 points for both) are presented here. To concatenate the data points within clusters, the average rates of deterioration were used as references, assuming the deck to be aged zero at condition index 100. For example, the bridge age for condition index 99 would be calculated using the average rate of deterioration between the indices 100 and 99. This is followed until condition index 0. For consecutive inspections within a cluster, the first inspection was marked on the plot with y-axis value equal to its health index, and x-axis value as the corresponding reference age. For the next inspection, y-axis value was its new condition index and x-axis value was its previous age plus the time since previous inspection. Eventually, a plot with condition index on the y-axis and age on the x-axis was obtained. An example for such a plot for one of the clusters is shown in Fig. 1, where the red cross-marks are individual data points.

In Phase 2, GPR was applied for fitting distributions of functions to individual clusters. A straight line was first fit to obtain prior mean values, followed by calculating posterior distribution of the functions. Exponential kernel function with characteristic length (l) equal to 45 and signal variance equal to 100 was found suitable for calculating the covariance matrix of the clusters. Posterior distribution for the cluster in Fig. 1 is shown in Fig. 2, where the red dotted line represents the posterior mean and the grey region shows the standard deviation. Fig. 3 displays the same technical plot for cluster 2. Certain ambiguities can be seen in these plots, for example the data points corresponding to different bridges in Fig. 2 and 3 are not exactly similar. This is due to the fact that only a single feature, i.e. the deck material, was used as the basis for identifying the clusters. The clustering step can be further improved, depending on the application, if more features are incorporated while clustering the bridges.

Summarising, the condition of a bridge deck, which has never been maintained and it is characterised by common features as the clusters, can be estimated based on its age using the plots in Fig. 2 and Fig. 3. The red line is the predicted value, and the grey region its confidence of prediction. The narrower the grey region, the more confident the prediction is and vice versa. As it can be observed from the plots, the presence of more and closely located data points causes more confident predictions. Moreover, for the conditions where we do not have any historical inspection data, the confidence of predictions is very low. For example, the standard deviation of predictions is very high after $t = 650$ in Fig. 2. In such situation, it is recommended for the infrastructure owner to resort to

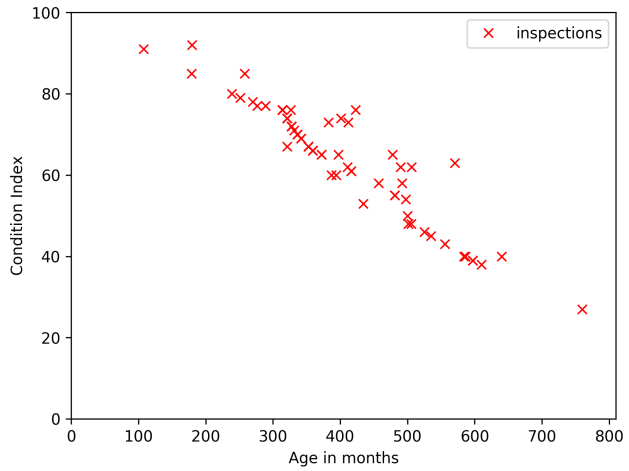

Fig. 1. Plot obtained after concatenating data points from cluster 1

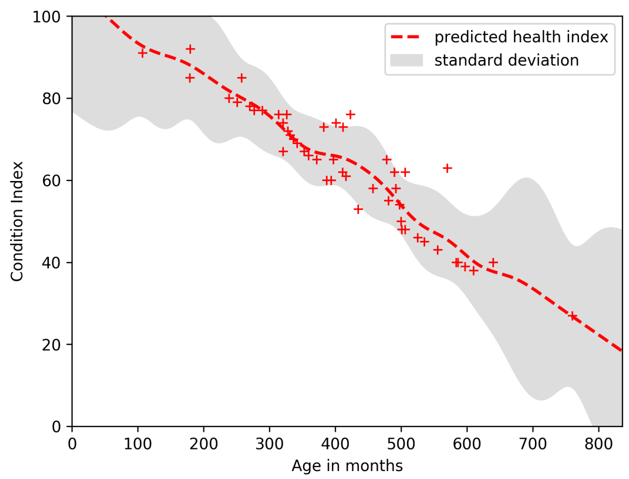

Fig. 2. Posterior distribution of functions for cluster 1

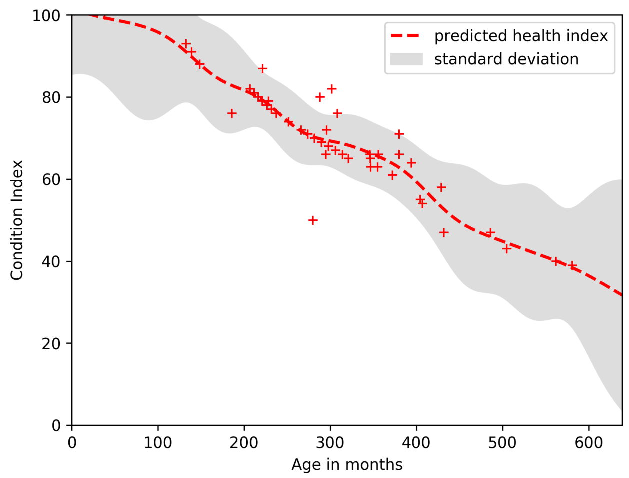

Fig. 3. Posterior distribution of functions for cluster 2

a conservative maintenance plan and observe the bridge deterioration more frequently.

5. CONCLUSIONS

The proposed collaborative GPR-based model was applied to a case study, with several significant outcomes being extracted. Firstly, most of the data from bridge inspections are concentrated within 80 to 40 condition index range. In the examined cluster, very few bridges have inspection records for very fragile conditions due to a safety threshold, set by the organisation. A bridge element must be repaired or replaced if its condition index is below 40. The repair or maintenance strategy is based on the type of element and existing defects. Secondly, the proposed method of clustering similar bridges to expand the dataset is deemed applicable since the rates of deterioration are nearly constant and the standard deviation tight within the cluster. Moreover, when GPR was applied for combined data of all bridges, the grey region was more spread out. Thirdly, there are some outliers, but GPR can understand the common behaviour and fit a distribution of predictor functions. It can be observed in both Fig. 2 and Fig. 3 that the grey region narrows down where the concentration of data points increases near the mean and broadens in the presence of the outliers indicating less confidence of the predictions.

The current paper proposed a methodology for modelling deterioration of bridges, for the situation where a single bridge does not have enough uninterrupted inspection records. The core idea is to expand the dataset by accumulating inspection records from similar bridges. The contribution of the presented work can be summarised as: the development of a methodology that can be applied to any type of bridge element, especially when the data is limited.

The current research can be synthesised with work already done in the same laboratory for predictive group maintenance of networks of bridges (Hadjidemetriou et al., 2020; Liang and Parlikad, 2020). Elements deterioration model is the first out of five phases for scheduling the optimal maintenance of bridge networks, under budget constraints. Thus, the proposed predictive model can be compared with the existing model (i.e. Markov chain-based) and replace it if the predictions are improved. More accurate predictive models can lead to improved maintenance decision-making and consequently safer bridges. This chain reaction continues with having a positive impact on infrastructure owners and bridge engineers regarding maintenance budget allocation, and finally terrestrial transport network users, whose safety and comfort will be enhanced. Additionally, prediction confidence, quantified by the proposed model, enables asset operators to undertake a safer risk-based maintenance strategy. In the sense that in case of a bridge being a critical transportation link, the operators would prefer a conservative approach for planning maintenance activities.

The clustering step however presents scope of development. If the bridges are clustered strictly (i.e. if all the features are required to be common), then it would cause the cluster size to diminish. In the extremity, this would be representative of the case where the bridges in a cluster are required to be identical and we would fall back to the original problem of data scarcity. On the other hand, lenient clustering would cause the bridges to be too different and undermine the purpose of clustering. Future work will focus on finding a trade-off between strict and lenient clustering. An automated algorithm will be developed to identify the sweet spot, where a cluster holds enough data to be modelled confidently but not diverse.

REFERENCES

ACSE (2017). Infrastructure Report Card. Technical report.

Alvarez, M.A., Rosasco, L., and Lawrence, N.D. (2011). Kernels for Vector-Valued Functions: a Review. *arXiv:1106.6251*. URL `http://arxiv.org/abs/1106.6251`.

Bocchini, P., Frangopol, D.M., and Deodatis, G. (2011). A random field based technique for the efficiency enhancement of bridge network life-cycle analysis under uncertainty. *Engineering Structures*, 33(12), 3208–3217. doi:10.1016/j.engstruct.2011.08.024.

Bocsi, B., Nguyen-Tuong, D., Csato, L., Scholkopf, B., and Peters, J. (2011). Learning inverse kinematics with structured prediction. In *2011 IEEE/RSJ International Conference on Intelligent Robots and Systems*, 698–703. Institute of Electrical and Electronics Engineers (IEEE). doi:10.1109/iros.2011.6094666.

Callow, D., Lee, J., Blumenstein, M., Guan, H., and Loo, Y.C. (2013). Development of hybrid optimisation method for Artificial Intelligence based bridge deterioration model - Feasibility study. *Automation in Construction*, 31, 83–91. doi:10.1016/j.autcon.2012.11.016.

Chan, C.Y., Huang, B., Yan, X., and Richards, S. (2010). Investigating effects of asphalt pavement conditions on traffic accidents in Tennessee based on the pavement management system (PMS). *Journal of Advanced Transportation*, 44(3), 150–161. doi:10.1002/atr.129. URL `http://doi.wiley.com/10.1002/atr.129`.

Chang, M., Maguire, M., and Sun, Y. (2019). Stochastic Modeling of Bridge Deterioration Using Classification Tree and Logistic Regression. *Journal of Infrastructure Systems*, 25(1), 04018041. doi:10.1061/(ASCE)IS.1943-555X.0000466.

Chapados, N. and Bengio, Y. (2008). Augmented Functional Time Series Representation and Forecasting with Gaussian Processes. In *Neural Information Processing Systems*.

Chen, F., Wei, D., and Tang, Y. (2010). Wavelet analysis based sparse LS-SVR for time series data. In *Proceedings 2010 IEEE 5th International Conference on Bio-Inspired Computing: Theories and Applications, BIC-TA 2010*, 750–755. doi:10.1109/BICTA.2010.5645219.

Chuang, T.Y., Perng, N.H., and Han, J.Y. (2019). Pavement performance monitoring and anomaly recognition based on crowdsourcing spatiotemporal data. *Automation in Construction*, 106, 102882. doi: 10.1016/j.autcon.2019.102882.

Duvenaud, D.K. (2014). Automatic Model Construction with Gaussian Processes. *Thesis*.

Estes, A.C. and Frangopol, D.M. (1999). Repair optimization of highway bridges using system reliability approach. *Journal of Structural Engineering*, 125(7), 766–774. doi:10.1061/(asce)0733-9445(1999)125:7(766).

Eurostat (2017). Passenger Transport Statistics. Technical report.

Frangopol, D.M., Lin, K.Y., and Estes, A.C. (1997). Life-Cycle Cost Design of Deteriorating Structures. *Journal of Structural Engineering*, 123(10), 1390–1401. doi: 10.1061/(ASCE)0733-9445(1997)123:10(1390).

Galal Ali, G., Elsayegh, A., Assaad, R., and El-adaway, I. (2019). Artificial Neural Network Model for Bridge Deterioration and Assessment. In *CSCE Annual Conference*.

Hadjidemetriou, G.M. and Christodoulou, S.E. (2019). Vision- and Entropy-Based Detection of Distressed Areas for Integrated Pavement Condition Assessment. *Journal of Computing in Civil Engineering*, 33(3), 04019020. doi:10.1061/(ASCE)CP.1943-5487.0000836.

Hadjidemetriou, G.M., Vela, P.A., and Christodoulou, S.E. (2018). Automated Pavement Patch Detection and Quantification Using Support Vector Machines. *Journal of Computing in Civil Engineering*, 32(1), 04017073. doi: 10.1061/(ASCE)CP.1943-5487.0000724.

Hadjidemetriou, G.M., Xie, X., and Parlikad, A.K. (2020). Predictive Group Maintenance Model for Networks of Bridges. *Transportation Research Record: Journal of the Transportation Research Board*, 2674(4), 373–383. doi: 10.1177/0361198120912226.

Ivanová, E. and Masárová, J. (2013). Importance of Road Infrastructure in the Economic Development and Competitiveness. *ECONOMICS AND MANAGEMENT*, 18(2), 263–274. doi:10.5755/j01.em.18.2.4253.

Kleiner, Y. (2001). Scheduling Inspection and Renewal of Large Infrastructure Assets. *Journal of Infrastructure Systems*, 7(4), 136–143. doi:10.1061/(ASCE)1076-0342(2001)7:4(136).

Kotze, R., Ngo, H., and Seskis, J. (2015). *Improved Bridge Deterioration Models , Predictive Tools and Costs*. URL `https://trid.trb.org/view/1360951`.

Lee, J., Guan, H., Loo, Y.C., and Blumenstein, M. (2012). Refinement of Backward Prediction Method for Reliable Artificial Intelligence-Based Bridge Deterioration Modelling. *Advances in Structural Engineering*, 15(5), 825–836. doi:10.1260/1369-4332.15.5.825.

Liang, Z. and Parlikad, A.K. (2020). Predictive group maintenance for multi-system multi-component networks. *Reliability Engineering and System Safety*, 195, 106704. doi:10.1016/j.ress.2019.106704.

Malekjafarian, A., Martinez, D., and Obrien, E.J. (2018). The Feasibility of Using Laser Doppler Vibrometer Measurements from a Passing Vehicle for Bridge Damage Detection. *Shock and Vibration*. doi: 10.1155/2018/9385171.

Mobley, R.K. (2002). *An Introduction to Predictive Maintenance (Second Edition)*. Elsevier. doi:10.1016/B978-075067531-4/50018-X.

Morcous, G., Rivard, H., and Hanna, A.M. (2002). Modeling Bridge Deterioration Using Case-based Reasoning. *Journal of Infrastructure Systems*, 8(3), 86–95. doi: 10.1061/(ASCE)1076-0342(2002)8:3(86).

Palau, A.S., Bakliwal, K., Dhada, M.H., Pearce, T., and Parlikad, A.K. (2018). Recurrent Neural Networks for real-time distributed collaborative prognostics. In *2018 IEEE International Conference on Prognostics and Health Management, ICPHM 2018*. Institute of Electrical and Electronics Engineers Inc. doi: 10.1109/ICPHM.2018.8448622.

Puz, G. and Radic, J. (2011). Life-cycle performance model based on homogeneous Markov processes. *Structure and Infrastructure Engineering*, 7(4), 285–296. doi: 10.1080/15732470802532943.

Ranjith, S., Setunge, S., Gravina, R., and Venkatesan, S. (2013). Deterioration Prediction of Timber Bridge Elements Using the Markov Chain. *Journal of Performance of Constructed Facilities*, 27(3), 319–325. doi: 10.1061/(ASCE)CF.1943-5509.0000311.

Rasmussen, C.E. (2004). Gaussian Processes in Machine Learning. In *Lecture Notes in Computer Science*, volume 3176, 63–71. Springer, Berlin, Heidelberg. doi: 10.1007/978-3-540-28650-9_4.

Richardson, R.R., Osborne, M.A., and Howey, D.A. (2017). Gaussian process regression for forecasting battery state of health. *Journal of Power Sources*, 357, 209–219. doi:10.1016/j.jpowsour.2017.05.004.

Thompson, P.D. and Johnson, M.B. (2005). Markovian bridge deterioration: developing models from historical data. *Structure and Infrastructure Engineering*, 1(1), 85–91. doi:10.1080/15732470412331289332.

Wei, D. and Liu, H. (2013). An Adaptive-Margin Support Vector Regression for Short-Term Traffic Flow Forecast. *Journal of Intelligent Transportation Systems*, 17(4), 317–327. doi:10.1080/15472450.2013.771107.

Wellalage, N.K.W., Zhang, T., and Dwight, R. (2015). Calibrating Markov Chain–Based Deterioration Models for Predicting Future Conditions of Railway Bridge Elements. *Journal of Bridge Engineering*, 20(2), 04014060. doi:10.1061/(ASCE)BE.1943-5592.0000640.

Wu, D., Yuan, C., Kumfer, W., and Liu, H. (2017). A life-cycle optimization model using semi-markov process for highway bridge maintenance. *Applied Mathematical Modelling*, 43, 45–60. doi:10.1016/j.apm.2016.10.038.

Zhang, W. and Wang, N. (2017). Bridge network maintenance prioritization under budget constraint. *Structural Safety*, 67, 96–104. doi:10.1016/j.strusafe.2017.05.001.